



Theory of Index Numbers

Price Statistics Division

Ministry of Statistics & Programme Implementation



Overview

- Concepts & Definitions of Index Numbers.
- Use of Index Numbers.
- Concept of Price
- Comparison of different Index Formulae
- Economic Interpretations of Price Indices



Definition

“Index numbers are specialized averages which are designed to measure the relative change in a group of related variables with respect to time, geographical location or other characteristics such as income, profession etc.”



Uses of Index Numbers

- Help in framing suitable economic policies
- Reveal trends and tendencies
- Use in forecasting future economic activities
- Use as a deflator/Inflator

Classification of Index Numbers

- Price Index-e.g. WPI/PPI,CPI etc.
- Quantity/Volume Index- e.g. IIP, IAP
- Value Index - e.g. Import & Export Index
- Special Purpose Index – e.g. Stock Market Index



Prices

“Price is an important economic variable in a market economy. Basic price data enter into the calculation of price index numbers. In other words, Index Numbers are the means of aggregation of these prices and work as indicator of price behaviour/movement in the economy.”



Price Index Numbers

A price index compares the prices of a “*set of products*” (Basket) at different points in time, or at different locations. It, therefore, measures *price changes* or price differentials rather than *price levels*. A price index shows how much must be paid for a set of products at some point in time relative to what would have been paid for the same set of products at another point in time, which latter is taken as the *reference/base of the comparison*.

Price Indices

Price Collection

- Production Floor
- Consumer Floor

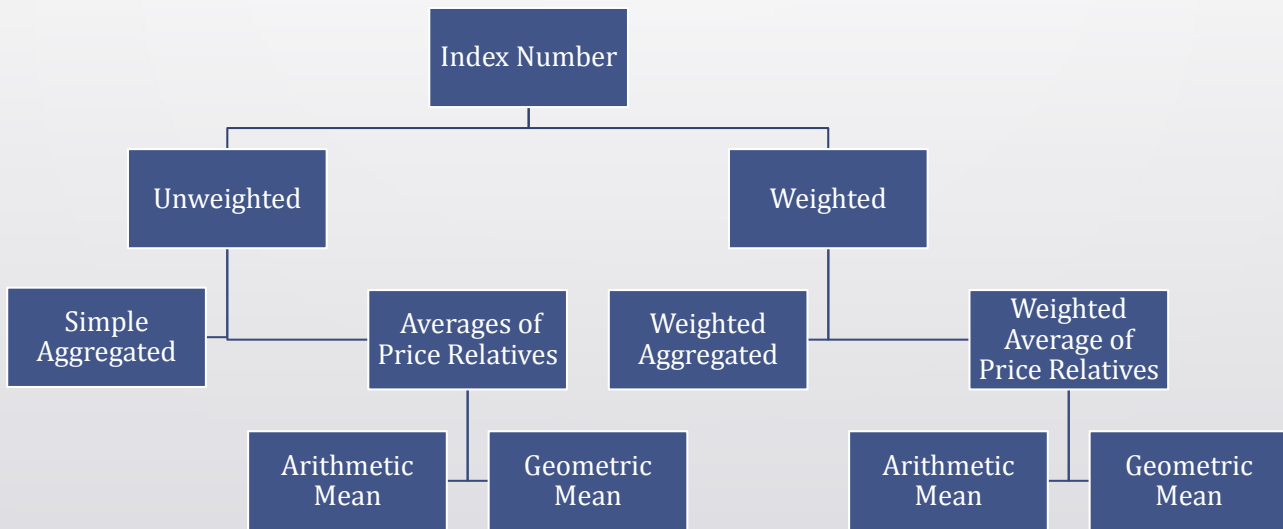
Aggregation of Prices

- Elementary Aggregates (Index No.)
- Major Aggregates (Index No.)

Inflation

- Interpretation/ Analysis of Prices Indices
- Use of Price Indices

Methods of Construction of Price Index Numbers





Stages of Price Index Compilation

The Price Indices are compiled at two levels;

1. Elementary (Low-Level) Aggregates
2. Major (Higher-Level) Aggregates

Elementary Aggregates Level

- The first stage in compilation of price indices is compilation of lower-level (elementary) price indices i.e. item level indices, which are then aggregated to obtain higher-level price indices.
- The item level index is calculated either on different varieties of same item or on different outlets of prices of same item. The weights are not, generally, available below the elementary aggregate level.
- Four most widely used elementary aggregate index formulae are *Mode Method, Dutot, Carli and Jevons*.

Methods of Elementary Aggregate

- Four methods are, generally, used for aggregating price of different item specifications/varieties into single item price;
- **Mode Method-** Most Popular specifications/varieties

- **Dutot's Method -**
$$P_{Du}^{0t} = \frac{\frac{1}{m} \sum_i p_i^t}{\frac{1}{m} \sum_i p_i^0} \times 100$$

Methods of Elementary Aggregate

- **Carli's Method -**
$$P_{Ca}^{0t} = \frac{1}{m} \sum_i \frac{p_i^t}{p_i^0} \times 100$$

- **Jevon's Method -**
$$P_{Je}^{0t} = \sqrt[m]{\prod_{i=1}^m \left(\frac{p_i^t}{p_i^0} \right)} \times 100$$

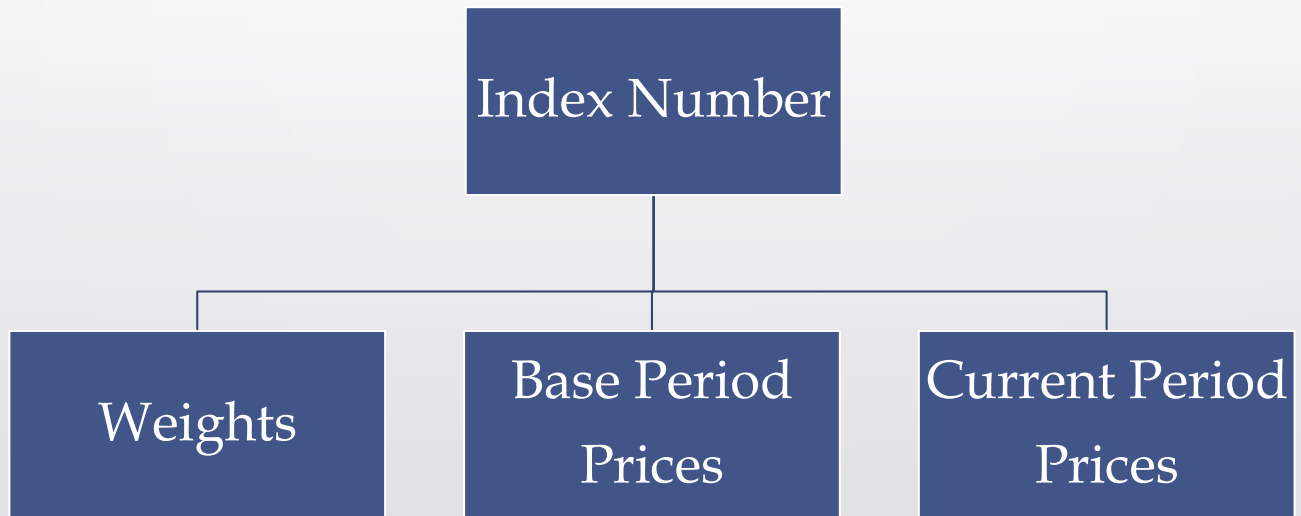
Table 1.1: Illustration for Calculating Elementary Aggregate:

S l. No.	Item Specification	P r i c e		P r i c e R e l a t i v e (P ¹ /P ⁰)	
		Base Period (P ⁰)	Current Period (P ¹)		
1.	Medicine Allopathic	1	2	3	
	i) Diabetes - Tab	29.00	55.40	1.9103	
	ii) Blood Pressure - Tab	23.00	42.44	1.8452	
	iii) Crocin Tablets, Strip	6.07	15.46	2.5470	
	iv) Benadryl - Cough Syrup	36.20	38.33	1.0588	
	v) Dettol Liquid Bottle	9.86	11.45	1.1613	
Arithmetic Mean of Prices		20.8260	32.6160	Dutot	1.5661
Geometric Mean of Prices		17.6373	27.5783	Jevon	1.6165
Arithmetic Mean of Price Relatives				Carli	1.7045
Geometric Mean of Price Relatives				Jevon	1.6165

Major (Higher-Level) Aggregates

- After getting lower-level (elementary) price indices i.e. item level indices, which are then aggregated to obtain higher-level price indices.
- The weights are, generally, used for combining elementary level aggregates (item level indices) into higher-level (sub-group/group/national level) aggregates.
- The most widely used higher level aggregate formulae are *Laspeyres, Lowe and Young*.

Components of Weighted Index Number



Weighted Aggregative Method

In this method appropriate weights are assigned to various commodities to reflect their relative importance in the group. Usually the quantities consumed, sold or marketed in the base year or in a given year are used as weights. If w_i is the weight attached to i^{th} commodity then the weighted price index is given by

$$P^{0t} = \frac{\sum_i (w_i \times p_i^t)}{\sum_i (w_i \times p_i^0)} \times 100 \quad \text{----- (4)}$$

Where p_i^t = price for i^{th} item in the current period 't'.

p_i^0 = prices for the base period '0'.

Test of Adequacy of Index Number Formulae/Methods

1. Unit Test

2. Time Reversal Test

$$P^{0t} \times P^{t0} = 1$$

3. Factor Reversal Test

$$P^{0t} \times Q^{0t} = \left[\frac{\sum_i (q_i^0 \times p_i^t)}{\sum_i (q_i^0 \times p_i^0)} \right] \times \left[\frac{\sum_i (q_i^t \times p_i^0)}{\sum_i (q_i^0 \times p_i^0)} \right] = \frac{\sum_i (q_i^t \times p_i^t)}{\sum_i (q_i^0 \times p_i^0)}$$

4. Circular Test

$$P^{01} \times P^{12} \times P^{20} = 1$$

Laspeyres' Method

Laspeyres Price Index is obtained by replacing ' w_i ' in the index formula by q_i^0 the quantity consumed/produced in base year which is given below:

$$P_{La}^{0t} = \frac{\sum_i (q_i^0 \times p_i^t)}{\sum_i (q_i^0 \times p_i^0)} \times 100$$

■ Its operational form which is used for compilation;

$$P_{La}^{0t} = \frac{\sum_i (q_i^0 \times p_i^0) \frac{P_i^t}{P_i^0}}{\sum_i (q_i^0 \times p_i^0)} \times 100$$

Laspeyres' Method (Contd./-)

$$P_{La}^{0t} = \sum_i w_i^0 \frac{P_i^t}{P_i^0} \times 100$$

Where $w_i^0 = \frac{(q_i^0 \times p_i^0)}{\sum_i (q_i^0 \times p_i^0)}$, denotes the share of expenditure/value of output of i^{th} item in the base period '0' and is termed as the weight the i^{th} item and $\frac{P_i^t}{P_i^0}$ is Price Relative of i^{th} item, which is the ratio of the current period 't' price to base period '0' price.

Paasche's Method

Paasche Price Index is obtained by replacing ' w_i ' in the index formula by q_i^t the quantity consumed/produced in current period 't' which is given below:

$$P_{Pa}^{0t} = \frac{\sum_i (q_i^t \times p_i^t)}{\sum_i (q_i^t \times p_i^0)} \times 100$$

Its operational form which may be used for compilation of price index;

$$P_{Pa}^{0t} = \frac{\sum_i (q_i^t \times p_i^t)}{\sum_i (q_i^t \times p_i^t) \frac{p_i^0}{p_i^t}} \times 100$$



Limitations:

- i. The Laspeyre's index shows an upward bias (in relation to the true cost of living index) with the passage of time i.e. 'overestimation' while the Paasche's index shows a downward bias i.e. 'underestimation' .
- ii. While using Paasche's method, it is necessary to collect the information on quantity as well as prices of current period which is very difficult to implement.

Dorbish and Bowley's Method

Dorbish and Bowley have suggested simple arithmetic mean of the two indices - Laspeyre's and Paasche's - so as to take into account the influence of both the periods i.e. current as well as base periods. The formula for constructing the index is :

$$P_{DB}^{0t} = \frac{1}{2} \left[\frac{\sum_i (q_i^0 \times p_i^t)}{\sum_i (q_i^0 \times p_i^0)} + \frac{\sum_i (q_i^t \times p_i^t)}{\sum_i (q_i^t \times p_i^0)} \right] \times 100$$

Fisher's Method

To overcome the limitations mentioned for earlier methods Irving Fisher constructed his 'Ideal Index' as the geometric mean of the Laspeyre's and Paasche's Indices, since the geometric mean would lie in between the two and hence would be nearest to the true cost of living index. Thus, Fisher's Ideal Index would be given by

$$P_{Fi}^{0t} = \sqrt{\left[\frac{\sum_i (q_i^0 \times p_i^t)}{\sum_i (q_i^0 \times p_i^0)} \times \frac{\sum_i (q_i^t \times p_i^t)}{\sum_i (q_i^t \times p_i^0)} \right]} \times 100$$



Fisher's Method

Fisher's Price Index number is known as 'Ideal' due to the following reasons:-

1. It is free from bias, since the upward bias of Laspeyres' index number is balanced to a great extent by the downward bias of Paasche's index number
2. It is based on the geometric mean, theoretically which is considered to be the best average for constructing index numbers.
3. It conforms to certain tests (3 out of 4 tests) of consistency.
4. This formula takes into account the influence of the current as well as the base year.

Marshall-Edgeworth's Method

Marshall-Edgeworth's Index is obtained by replacing W in the index formula by the average quantity of base year and current year i.e. $(q_i^0 + q_i^t)/2$, which is given below :

$$P_{ME}^{0t} = \frac{\sum_i p_i^t (q_i^0 + q_i^t)}{\sum_i p_i^0 (q_i^0 + q_i^t)} \times 100$$

Lowe's Method

In this method appropriate fixed weights, not necessarily of base year or current year, are assigned to various commodities to reflect their relative importance. If q_i^a is the weight attached to a commodity then the index is given by

$$P_{Lo}^{a0t} = \frac{\sum_i (q_i^a \times p_i^t)}{\sum_i (q_i^a \times p_i^0)} \times 100$$

Where q_i^a may belong to any period, including one between 0 and t, however, generally it exists before period 0. It means *weight reference period 'a'* differ from *price reference periods either '0' or 't'*. The operational form of Lowe Method, which may be used for compilation of price indices, is as follow;

$$P_{Lo}^{a0t} = \frac{\sum_i (q_i^a \times p_i^0) \frac{P_i^t}{P_i^0}}{\sum_i (q_i^a \times p_i^0)} \times 100$$

Young's Method

Instead of holding constant the quantities of period 'a', a statistical office may calculate CPI as a weighted arithmetic average of the individual price relatives, holding constant the revenue shares of period 'a'. The resulting index is called a Young method. The Young method is defined as follows:

$$P_{Yo}^{a0t} = \frac{\sum_i (q_i^a \times p_i^a) \frac{p_i^t}{p_i^o}}{\sum_i (q_i^a \times p_i^a)} \times 100$$

i.e.

$$P_{Yo}^{a0t} = \sum_i w_i^a \frac{p_i^t}{p_i^o} \times 100$$

The Lowe index may be preferred to the Young index because the Young index has some undesirable properties that cause it to fail some tests of adequacy for being a valid index number.



General Remarks

- In practice, the quantities/expenditures used for compilation of price indices, particularly for CPI, usually has to be based on a household consumption expenditure survey, generally conducted in an earlier period than either of the two periods whose prices are compared. The *price reference period '0'* is usually later than the *weight reference period 'a'* because of the time needed to collect and process the household consumption expenditure data. *Consequently, Lowe indices are widely used for CPI purposes.*
- An overwhelming majority of the index numbers of consumer prices in vogue in different countries of the world are based on the Laspeyre's formula as it is simple to calculate and requisite data is easily available.



Fixed Base Indices
vis-à-vis
Chain Base Indices